

文章编号 1004-924X(2009)06-1409-06

不同质量块布局双端固支梁的等效弹簧系数的热效应

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摘要:通过理论研究和有限元仿真分析了热效应对两种不同质量块布局的双端固支梁的等效弹簧系数的影响,两种质量块分别布局为质量块与梁对称布置和质量块位于梁平面一侧布置。推导了双端固支梁等效弹簧系数随温度的变化公式,仿真分析了双端固支梁-质量块系统的一阶固有频率以及质量块在静载荷作用下的位移。通过固有频率及载荷-位移两种方式,分别得到了两种布局下的质量系统在不同温度下的等效弹簧系数。对于质量块对称布置和单侧布置,根据一阶固有频率得到的等效弹簧系数随温度的变化与理论分析都吻合;而利用载荷-位移得到的等效弹簧系数随温度的变化,需要对位移进行修正。计算结果表明,对于质量块位于梁一侧的系统,温度变化对其等效弹簧系数有重要影响,质量块的不对称会在温度变化时产生对梁的弯矩作用,从而使质量块产生位移,造成微机电传感器和执行器输出信号的改变。

关键词:热效应;等效弹簧系数;双端固支梁;质量块布置

中图分类号:TP212.12 **文献标识码:**A

Thermal effect on equivalent spring constants of double-clamped beams with different mass arrangements

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Abstract: Thermal effect on the equivalent spring constants of double-clamped beams (bridges) with two kinds of mass arrangements including central mass and side mass were studied by theoretical analysis and finite element simulation. An equation to describe the change of the equivalent spring constants of beams due to temperature variation was derived and the equivalent spring constants from simulation results of the fundamental vibration frequency and the displacement under a static load were obtained respectively by the finite element analysis. Results show that the temperature variation has an important effect on the equivalent spring constants. For the fundamental vibration frequency, there are a good agreement between the derived equation and the vibration simulation results for the both structures. For the displacement under a static load, the asymmetry in mass arrangement results in the bending of the beam when temperature shifts, and the bending causes a displacement of the mass to the side. As a result, the asymmetric arrangement of mass changes the initial displacement of mass

Received date:2008-12-24;**Revised date:**2009-04-30.

Foundation item:Supported by the National Natural Science Foundation of China (Grant No. 50730009)

when temperature varies, which would cause the changes of output signals for Micro Electro Mechanic System (MEMS) sensors or actuators.

Key words: thermal effect; equivalent spring constant; double-clamped beam; mass arrangement

1 Introduction

A structure of two clamped beams supporting a mass is often used for micro inertial sensors (such as accelerometers and gyroscopes), resonators and other micro actuators^[1-2]. Quad-beam structures, another structures usually found in MEMS^[3], can be also seemed as double-clamped beam-mass structures. If the loading of a beam-mass structure is in the z -direction, the displacement of the mass is piston-like because of the symmetry of the structure in the x - and y -direction. Moreover, a double-clamped beam will endure thermal stress when temperature changes^[4]. When the temperature rises up, the beam becomes loose, while the drop of temperature will stiffen the beam. If the beam is considered as a spring, the thermal stress can affect the equivalent spring constant and even make the beam buckling^[4]. Therefore the sensitivities of the sensors with bridge-mass structures will change when temperature varies. In most of the cases with MEMS structures, beam-mass configurations with side mass are employed, partly for the fabricating convenience. But this configuration is asymmetric in z -direction, which may introduce thermal deformation.

In this paper we present a detailed analysis of the thermal effect on the equivalent spring constant of double clamped beam-mass structures with two different arrangement of mass, including structures with central mass and side mass. Firstly, an equation describing the equivalent spring constant at different temperature is deduced. Secondly, finite element analysis is conducted to get the equivalent spring constant

through vibration frequency and static displacement under inertial loads for verifying the theoretical equation. Then we compare the variation of equivalent spring constant between the two structures and discuss the cause inducing the difference of equivalent spring constants.

2 Theory

For a double clamped beam with an axial force, considering the vibration of small amplitude, the fundamental vibration frequency of the beam can be written as follows^[5]:

$$f(N) = f(0) \cdot \sqrt{1 + \frac{0.2949NL^2}{12EI}}, \quad (1)$$

where $f(N)$ is the frequency with the axial force N , $f(0)$ is the fundamental vibration frequency without axial force, L is the beam length, E is the Young's modulus of the beam, and I is the moment of inertia of the beam cross section. If the beam is considered as a spring, according to the vibration theory, the spring constant is:

$$k = 4\pi^2 m f^2, \quad (2)$$

where k is the equivalent spring constant, m is the equivalent mass, f is the fundamental vibration frequency.

For a double-clamped beam with a mass as shown in Fig. 1 or Fig. 2, since the mass is much

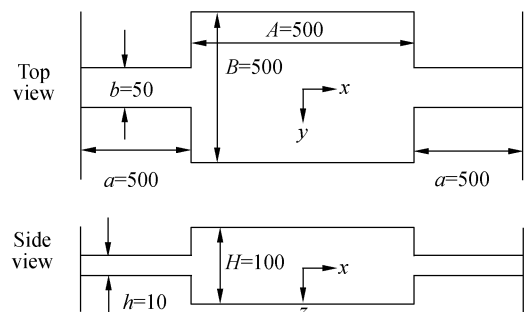


Fig. 1 Beam-mass structure with a central mass (structure 1)

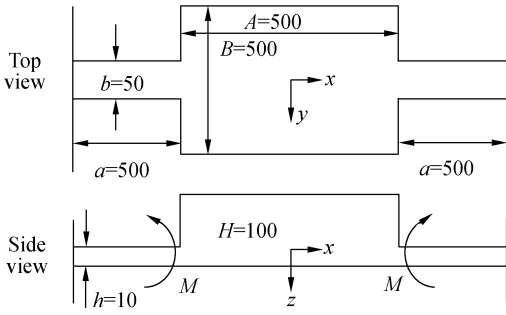


Fig. 2 Beam-mass structure with a side mass (structure 2)

wider and/or thicker than the beam so that the bending of the mass can be negligible. And the equation (2) can also be applicable for these structures while m represents the equivalent mass of the structures. The k can also be calculated from the displacement under loading by:

$$k = \frac{\Delta F}{\Delta z}, \quad (3)$$

where ΔF is loading in z -direction, Δz is the displacement of the mass in z -direction.

In this work, small temperature range is considered, so we assume that the Young's modulus is not change with temperature. If temperature changes ΔT , the axial force N induced by thermal stress is given by $N = EA\alpha\Delta T$, where A is the area of the beam cross section, α is the coefficient of thermal expansion. For structures shown in Fig. 1 or Fig. 2, the axial force applied to the beam includes two parts, the axial forces induced by the beam and the mass respectively. So the total axial force applied on the beam can be approximately given by:

$$N_{\text{total}} = N_{\text{beam}} + N_{\text{mass}} = (E_{\text{beam}} + E_{\text{mass}}) \cdot A \cdot \alpha \cdot \Delta T, \quad (4)$$

We define β as the coefficient of the equivalent spring constant by:

$$\beta(N) = \frac{k(N)}{k(0)}, \quad (5)$$

where $k(N)$ is the spring constant with axial force N , $k(0)$ is the spring constant without axial force. From equations (1), (4) and (5), we have the equation:

$$\beta(\Delta T) = \frac{k_{\text{beam}}(\Delta T)}{k_{\text{beam}}(0)} = 1 + \frac{0.2949(E_{\text{beam}} + E_{\text{mass}})A\alpha \cdot \Delta T \cdot L^2}{12E_{\text{beam}}I}, \quad (6)$$

3 Simulation and discussion

Consider two double-clamped beam-mass silicon structures shown in Fig. 1 and Fig. 2. The only difference between the two structures is that the mass of the structure 2 is placed on top of the beam while the mass of the structure 1 is symmetric in the z -direction. The thickness and the width of the masses are much larger than those of the beams, so the bending of the masses can be negligible. Nastran software was used to simulate the normal mode and static displacements. The finite element models using 3D Hexa elements are shown in Fig. 3. The reference temperature in this simulation is set as 20 °C, at which it is assumed that there is no thermal stress. Firstly, we use Nastran to get the frequency of the structure's up-and-down vibration mode at different temperature, and the equivalent spring constant k can be calculated using the equation (2). Then we get β from the equation (5). Secondly, we simulated the displacement of the structures under an inertial load of 10 times of gravity when changing the structure's temperature, and get the displacement of one point at the center of the interface between the beam and the mass. Then k can be got from the equation (3) and β is calculated by the equation (5).

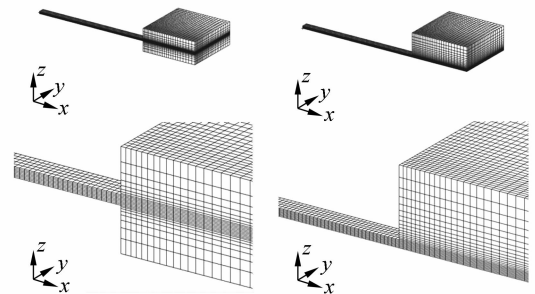


Fig. 3 Three dimensional FEM models of the structures

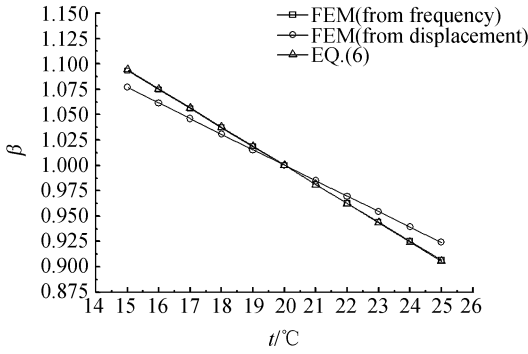


Fig. 4 β changes with temperatures (structure 1)

Fig. 4 and Fig. 5 show β changing with temperature for structure 1 and structure 2 respectively. From Fig. 4, we find that the β got from the frequency is consistent with Eq. (6), and the β from the displacement is close to Eq. (6) with relative error less than 2% in the considered temperature range ($t = 15 \sim 25 \text{ }^\circ\text{C}$). This shows that the theory is accurate to predict the variation of equivalent spring constant due to the changing of temperature for structure 1. In Fig. 5, the β got from the displacement of structure 2 increases with temperature rising, which is very different from the β from the frequency and Eq. (6). To find the cause of the difference, additional simulations were conducted to get the displacement of the mass with the temperature variation without loading and the x -component stress distribution at the beam ends interfaced with mass. The displacement contours of the two structures at $25 \text{ }^\circ\text{C}$ are shown in Fig. 6. It is found that the beam of structure 2 is bending

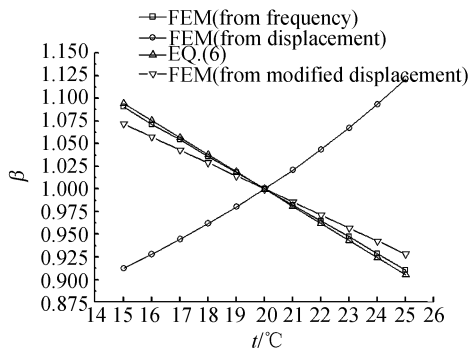


Fig. 5 β changes with temperatures(structure 2)

up, while the beam of structure 1 remains flat. Fig. 7 illustrates x -component stress distributions at the beam end interfaced with mass. The x -component stress distribution of structure 1 is symmetric in the z -direction, so there is no bending moment. The asymmetric stress distribution of structure 2 in the z -direction leads bending moment. Fig. 8 shows the displacements of mass induced by temperature variation and inertial load respectively as well as the total displacements. In Fig. 8, the red circles represent total displacement under inertial load while temperature changes, and the blue squares show the displacement without load when temperature shifts. By removing the displacement induced by asymmetric mass-arrangement from the total displacement, we obtained the modified displacement only induced by inertial load (shown by black triangles in Fig. 8). Then β from the modified displacement is close to Eq. (6) with relative error less than 2.5%, as shown in Fig. 5.

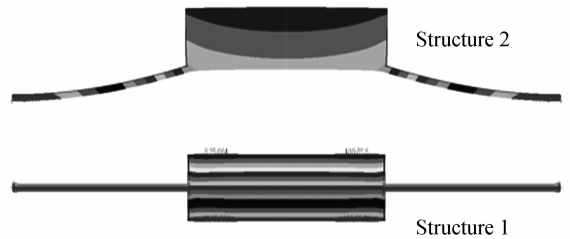


Fig. 6 Schematic graph of displacement contour (no mechanical loads, $t = 25 \text{ }^\circ\text{C}$)

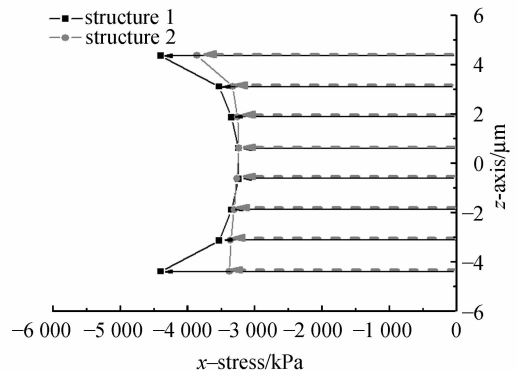


Fig. 7 X -component stress distributions at beam end interfaced with mass (no mechanical loads, $t = 25 \text{ }^\circ\text{C}$)

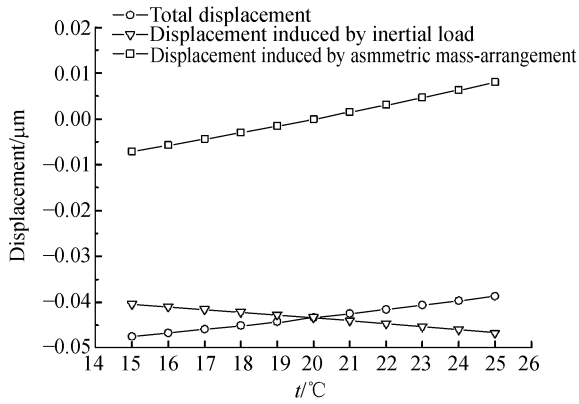


Fig. 8 Displacements changes with temperatures under different loads (structure 2)

The analysis results show that the mass arrangement can affect the equivalent spring constant. If the mass is arranged symmetric in z -direction, Eq. (6) can precisely depict the change of beam's equivalent spring constant due to the temperature variation. When the mass is placed on side (top or bottom) of the beam, the asymmetric arrangement in z -direction brings a thermal bending of the beams and causes a thermal displacement of mass to the side at which the

mass is placed. This changes the total displacement under load. For an accelerometer using sensitive structure similar as the structure 2, a change of temperature of the sensor will cause a change of the output signal and this is a source of error for the sensor.

4 Conclusions

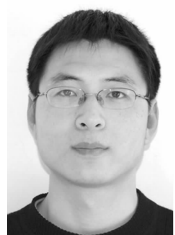
The thermal effect on the equivalent spring constant of double clamped beam with mass has been analyzed. We find that the temperature has an important effect on the equivalent spring constant of the bridge beams with central mass (symmetric in the z -direction) than that with side mass (asymmetric in the z -direction). For a structure with side mass which is widely used in MEMS sensors, asymmetric arrangement of mass in the z -direction will change the initial displacement of mass when temperature shifts, and this causes change of the output signals.

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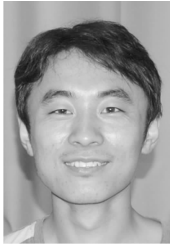
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● 下期预告

小波变换和 Zernike 矩的图像区域复制篡改鲁棒取证

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现有的图像区域复制篡改检测算法其后处理的鲁棒性大多比较差, 并且时间复杂度高。针对现有篡改技术, 提出了一种有效且快速的检测与定位篡改区域算法。该算法首先将图像进行小波分解, 对低频图像进行块分解, 提取每块的 Zernike 矩特征, 并将特征向量排序, 然后为每个特征向量搜索阈值符合的相似特征向量, 最后利用区域面积阈值去除错误的相似块对, 并结合数学形态学定位篡改区域。实验结果表明该算法不仅能有效地对抗常规的后处理操作, 而且只在 $1/4$ 图像上搜索块空间, 提高了运算效率。本方法可用于面向图像内容的真实性鲁棒取证。